

Name: _____

Date: _____

M12H Section 4.6A Solving Trigonometric Equations with Identities:

1. Given each equation below, indicate which of the following are Identities and which ones are not. State the ones that are false::

a) $\sin(a+b) = \sin a + \sin b$	b) $\cos(-x) = \cos(x)$	c) $\sin(-x) = -\sin(x)$
d) $\sin 2x = 2\sin x$	e) $\frac{\sin 2\theta}{2} = \sin \theta$	f) $\sin 2\theta = 2\sin \theta \cos \theta$
g) $\tan \theta = \frac{\cos \theta}{\sin \theta}$	h) $\tan \theta = \frac{\sin \theta}{\cos \theta}$	i) $\csc \theta = \frac{1}{\cos \theta}$
j) $\sin^2 \theta + \cos^2 \theta = 1$	k) $\cos^{-1}(\theta) = \frac{1}{\cos \theta}$	l) $\sec \theta = \frac{1}{\cos \theta}$
m) $\csc(-\theta) = -\frac{1}{\sin \theta}$	n) $\sin(a+b) = \sin a \cos b + \sin b \cos a$	o) $\tan \theta = \frac{\sin \theta}{\cos \theta}$
p) $\sin 2\theta = 1 \rightarrow \sin \theta = \frac{1}{2}$	q) $\sin(a-b) = -\sin(b-a)$	r) $\sin(\pi - \theta) = \sin \theta$
s) $\cos 2\theta = 1 - 2\sin^2 \theta$	t) $\cos 4\theta = 1 - 2\sin^2 2\theta$	u) $\sin \theta = \cos\left(\frac{\pi}{2} - \theta\right)$
v) $\sin(-\theta) + d = -\sin \theta + d$	w) $\cos(a-b) = \cos a \cos b + \sin a \sin b$	x) $\cos(a-b) = \cos a - \cos b$
y) $\sec(-\theta) = \sec \theta$	z) $\tan\left(\frac{a}{b}\right) = \frac{\tan a}{\tan b}$	aa) $\tan(a-b) = \frac{\sin(a-b)}{\cos(a-b)}$
Ab) $\cos^{-1} \theta = \frac{adj}{hyp}$	Ac) $\cos^{-1}\left(\frac{adj}{hyp}\right) = \theta$	Ad) $1 + \cot^2 \theta = \csc^2 \theta$

2. Solve the following equations for $0 \leq \theta \leq 2\pi$. Use the trigonometric Identities shown in class. Make sure to check for extraneous roots: Show all your work and steps:

i) $\cos 2\theta = \cos^2 \theta$

ii) $\cos \theta \sin \theta = \frac{1}{4}$

iii) $\sin 2\theta = 2 \cos \theta$

iv) $\cos \theta \cot 2\theta = \cos \theta$

v) $\cos \theta \tan \theta = \cos \theta$

vi) $\cot \theta = 2 \sin 2\theta$

Vii) $\cos 2\theta - \cos \theta = 0$

Viii) $\cos(\theta + \pi) - \cos^2 \theta = 0$

ix) $\sin^2 \theta + 2 \cos^2 \theta = \frac{7}{4}$

x) $3 - 3 \csc \theta + \cot^2 \theta = 0$

$$\text{xi)} \quad 2 \sin^2 \theta - \cos \theta - 1 = 0$$

$$\text{xii)} \quad 2 \sin^2 \theta \cot \theta - 5 \sin \theta \cot \theta - 3 \cot \theta = 0$$

$$\text{xiii)} \quad 3 \cos^2 \theta - 8 \cos \theta + 4 = 0$$

$$\text{si)} \quad 6 \sin \theta (\tan \theta) - \tan \theta - \sec \theta = 0$$

$$\text{sii)} \quad \cot \theta = \sin 3\theta$$

$$\text{siii)} \quad 8 \cot \theta \cos \theta - 6\sqrt{3} \cot \theta + 3 \csc \theta = 0$$

3. Find all the NPV(s) for each of the following expressions:

$$\text{i)} \quad \frac{\sec \theta}{2 \sin \theta + 1}$$

$$\text{ii)} \quad \frac{\sin \theta \sec \theta}{\cot \theta}$$

$$\text{iii)} \quad \frac{\sec^2 \theta - 1}{1 + \tan^2 \theta}$$

$$\text{iv)} \quad \frac{\cos \theta}{1 + \sin \theta} + \frac{\cot \theta}{1 - \sin \theta}$$

$$\text{v)} \quad \frac{1 + \cos^2 \theta - \sin \theta}{\sin \theta \cos \theta \tan \theta}$$

$$\text{vi)} \quad \frac{\cos 2\theta - 1}{3 \sin 2\theta - 2}$$

$$\text{vii)} \quad \cot \theta (\tan \theta) (\csc \theta) \sin \theta$$

$$\text{viii)} \quad \frac{1 - \cot \theta}{1 + \tan \theta} \times \frac{\sin \theta}{\sec \theta}$$

4. Prove the following Identities. Also indicate any restrictions on the domain:

i) $\csc \theta \sin 2\theta - \sec \theta \cos 2\theta = \sec \theta$

ii) $\frac{\sin \theta}{1 - \sin \theta} + \frac{\sin \theta}{1 + \sin \theta} = 2 \sin 2\theta \sec^3 \theta$

iii) $\frac{\tan \theta (\cos \theta + \cot \theta)}{\sec \theta + \tan \theta} = \frac{\sin \theta \sin 2\theta}{2 - 2 \cos^2 \theta}$

iv) $\frac{\cos \theta + \sin \theta \tan \theta}{\sin \theta \sec \theta} = \csc \theta$

v) $\frac{\tan \theta}{\sec \theta + 1} = \frac{2 \cos \theta - 2 \cos^2 \theta}{\sin 2\theta}$

vi) $\frac{\sin 2\theta + \sin \theta}{1 + \cos \theta + \cos 2\theta} = \cot \left(\frac{\pi}{2} - \theta \right)$

vii) $(\sec \theta - \cos \theta)(\csc \theta - \sin \theta) = \frac{\tan \theta}{1 + \tan^2 \theta}$

viii) $\frac{\sin 4\theta - \sin 2\theta}{\cos 4\theta + \cos 2\theta} = \tan \theta$

ix) $\tan 2\theta = \frac{2}{\cot \theta - \tan \theta}$

x) $\frac{\sec \theta}{1 - \sin \theta} = \frac{1 + \sin \theta}{\cos^3 \theta}$